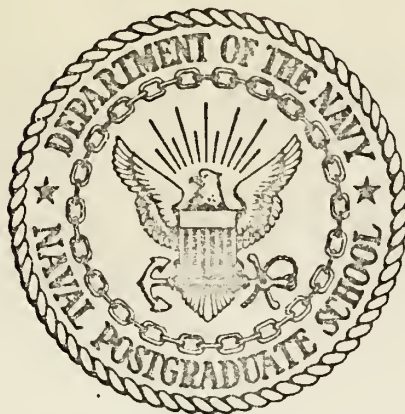


SOME ASPECTS OF THE BEHAVIOR OF
TWO COMPONENT PARALLEL SYSTEMS

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THESIS

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TWO COMPONENT PARALLEL SYSTEMS

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Some Aspects of the Behavior of
Two Component Parallel Systems

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ABSTRACT

The behavior of the system hazard rate function of a two component parallel system is investigated. The inter-relationships between the probabilities that the components composing the system are alive and the system hazard rate is examined with special attention to certain points where there are important changes in the behavior of the hazard rate function. The behavior of the system hazard rate function is shown to depend upon the rates of change of the probabilities that the components of the system are alive.

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I. INTRODUCTION

In the field of reliability, there has been an extensive study of the properties of the increasing hazard rate (IHR) and decreasing hazard rate (DHR) classes of life distributions. The exponential life distributions with their constant hazard rates form a dividing line between these two classes. It is reasonable to expect that the life distribution of redundant systems composed of devices with exponential life distributions fall into the IHR class. However, when two devices with different exponential life distributions are combined in a parallel redundant system, the system hazard rate is at first increasing and then decreasing. The underlying causes of this interesting behavior is the subject of the thesis.

The usual definition of the probabilistic hazard rate function of a device with a random lifetime T will be used throughout this thesis. The hazard rate $r(t)$ of a device is the density function $f(t)$ for the lifetime of the device divided by its survival function $\bar{F}(t)=P[T>t]$.

A distribution has an increasing hazard rate if $r(t)$ is monotone non-decreasing in t , and has a decreasing hazard rate if $r(t)$ is monotone non-increasing in t . References 1 and 2 contain an extensive survey of the properties of the IHR and DHR classes of life distributions. The exponential life distributions are included in both of these classes.

In Reference 3, Esary and Proschan gave sufficient conditions for a system to have an increasing hazard rate when it is composed of identical components with increasing hazard rates. A parallel system of two components, with identical exponential life distributions, satisfies these conditions and it has an increasing hazard rate. Esary and Proschan, also gave an example to show that when two components with different exponential life distributions form a parallel system, the system survival function need not have an increasing hazard rate. In this case the system hazard rate initially increases, overshoots and then decreases to an asymptotic value equal to the lowest of the two component failure rates.

In Section II, the derivation of the system hazard rate for a system of two parallel components having constant non-identical failure rates is reviewed. In Section III, a characterization of the system hazard rate as a function of some conditional state probabilities is presented and their interrelationships are discussed. In Section IV, an empirical approximation for the time at which the system hazard rate is a maximum is exhibited.

II. SYSTEM HAZARD RATE

Consider a system of two independently functioning components with exponential lifetimes T_1, T_2 and survival functions

$$\bar{F}_1(t) = e^{-\lambda_1 t}$$

$$\bar{F}_2(t) = e^{-\lambda_2 t}.$$

Assume without loss of generality that $\lambda_2 > \lambda_1$. The equations for the system survival function and the system hazard rate are generally known.

With the components in parallel, the system lifetime T has a survival function

$$\begin{aligned}\bar{F}(t) &= P[\max T_1, T_2 > t] \\ &= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}.\end{aligned}$$

The density of T is

$$\begin{aligned}f(t) &= -\frac{d}{dt} \bar{F}(t) \\ &= \lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} - (\lambda_1 + \lambda_2) e^{-(\lambda_1 + \lambda_2)t}.\end{aligned}$$

The system hazard rate is

$$\begin{aligned}r(t) &= \frac{f(t)}{\bar{F}(t)} \\ &= \frac{\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} - (\lambda_1 + \lambda_2) e^{-(\lambda_1 + \lambda_2)t}}{\bar{F}(t)}.\end{aligned}$$

Examining $r(t)$ at $t=0$ and as t approaches infinity gives the expected results;

$$r(0) = 0$$

$$\lim_{t \rightarrow \infty} r(t) = \min(\lambda_1, \lambda_2) = \lambda_1 \quad .$$

The derivative of $r(t)$ with respect to time is

$$\begin{aligned} r'(t) &= \frac{\bar{F}(t)f'(t) + f^2(t)}{\bar{F}^2(t)} \\ &= \frac{[\lambda_2^2 e^{-\lambda_1 t} + \lambda_1^2 e^{-\lambda_2 t} - (\lambda_1 - \lambda_2)^2] e^{-(\lambda_1 + \lambda_2)t}}{\bar{F}^2(t)} \quad . \end{aligned}$$

Looking at the sign of $r'(t)$, it is clear that the hazard rate is increasing if and only if

$$\lambda_2^2 e^{-\lambda_1 t} + \lambda_1^2 e^{-\lambda_2 t} > (\lambda_1 - \lambda_2)^2 \quad \text{and decreasing if and only if}$$

$\lambda_2^2 e^{-\lambda_1 t} + \lambda_1^2 e^{-\lambda_2 t} < (\lambda_1 - \lambda_2)^2$. If $\lambda_1 = \lambda_2$, $r(t)$ is

increasing for all $t > 0$. The system hazard rate must be at a maximum for the value of t which is a solution to

$$\lambda_2^2 e^{-\lambda_1 t} + \lambda_1^2 e^{-\lambda_2 t} - (\lambda_1 - \lambda_2)^2 = 0$$

III. SYSTEM HAZARD RATE IN TERMS OF STATE PROBABILITIES

If the parallel system is alive at some time $t > 0$, it exists in one of three possible states, i.e. both components alive, component one alive and component two dead, or component one dead and two alive. The system hazard rate and its derivative have simple expressions in terms of the conditional probabilities for these states and their derivatives. The probability that the system is in state i , $i=0,1,2$, given it is alive at a time t , will be called the state probability, denoted by $P_i(t)$. For a system with lifetime T and component lives T_1 and T_2 , the state probabilities are:

(1) both components alive

$$\begin{aligned} P_0(t) &= P[T_1 > t, T_2 > t | T > t] \\ &= \frac{P[T_1 > t, T_2 > t, \max(T_1, T_2 > t)]}{P[\max(T_1, T_2 > t)]} \\ &= \frac{e^{-(\lambda_1 + \lambda_2)t}}{\bar{F}(t)}, \end{aligned}$$

(2) component 1 alive and 2 dead

$$\begin{aligned} P_1(t) &= P[T_1 > t, T_2 < t | T > t] \\ &= \frac{P[T_1 > t, T_2 < t, \max(T_1, T_2 > t)]}{P[\max(T_1, T_2 > t)]} \end{aligned}$$

$$\begin{aligned}
P_1(t) &= \frac{e^{-\lambda_1 t} - e^{-(\lambda_1 + \lambda_2)t}}{\bar{F}(t)} \\
&= 1 - \frac{e^{-\lambda_2 t}}{\bar{F}(t)} \\
&= (e^{\lambda_2 t} - 1)P_0(t) ,
\end{aligned}$$

(3) component 2 alive and 1 dead

$$\begin{aligned}
P_2(t) &= P[T_1 < t, T_2 > t | T > t] \\
&= \frac{P[T_1 < t, T_2 > t, \max(T_1, T_2) > t]}{P[\max(T_1, T_2) > t]} \\
&= \frac{e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}}{\bar{F}(t)} \\
&= 1 - \frac{e^{-\lambda_1 t}}{\bar{F}(t)} \\
&= (e^{\lambda_1 t} - 1)P_0(t) .
\end{aligned}$$

It is clear that $P_0(t) + P_1(t) + P_2(t) = 1$, for all t .

Examining the state probabilities at $t = 0$ and as t approaches infinity produces the following results.

$$P_0(0) = 1, \quad \lim_{t \rightarrow \infty} P_0(t) = 0$$

$$P_1(0) = 0, \quad \lim_{t \rightarrow \infty} P_1(t) = 1$$

$$P_2(0) = 0, \quad \lim_{t \rightarrow \infty} P_2(t) = 0 .$$

At first glance, it might seem that $P_1(t)$ would equal $P_0(t)$ at the mean time to failure for component 2, but,
 $P_1(t) = P_0(t)$ if and only if

$$\frac{e^{-\lambda_1 t} - e^{-(\lambda_1 + \lambda_2)t}}{\bar{F}(t)} = \frac{e^{-(\lambda_1 + \lambda_2)t}}{\bar{F}(t)}$$

or

$$e^{-\lambda_1 t} = 2e^{-(\lambda_1 + \lambda_2)t};$$

or

$$1 = 2e^{-\lambda_2 t},$$

or

$$t = \frac{1}{\lambda_2} \ln 2.$$

Similarly, $P_2(t) = P_0(t)$ if and only if

$$t = \frac{1}{\lambda_1} \ln 2.$$

The probability that a particular single component is alive at some time t is greater than the probability that both are still alive for any time t greater than the mean time to failure of the other component multiplied by a constant $\ln 2$.

Further, for $\lambda_2 > \lambda_1$, $P_1(t) > P_2(t)$ for all t , since $P_1(t) > P_2(t)$ if and only if

$$1 - \frac{e^{-\lambda_2 t}}{\bar{F}(t)} > 1 - \frac{e^{-\lambda_1 t}}{\bar{F}(t)}$$

or

$$e^{-\lambda_2 t} < e^{-\lambda_1 t} ,$$

or

$$\lambda_2 > \lambda_1 .$$

The derivatives of the state probabilities with respect to time are:

$$P_0'(t) = \frac{\bar{F}(t)[-(\lambda_1 + \lambda_2)e^{-(\lambda_1 + \lambda_2)t} - e^{-(\lambda_1 + \lambda_2)t} \bar{F}'(t)]}{\bar{F}^2(t)}$$

$$= \frac{e^{-(\lambda_1 + \lambda_2)t}}{\bar{F}(t)} \left[\frac{f(t)}{\bar{F}(t)} - (\lambda_1 + \lambda_2) \right]$$

$$= P_0(t)[r(t) - (\lambda_1 + \lambda_2)] .$$

$$P_1'(t) = - \frac{\bar{F}(t)(-\lambda_2 e^{-\lambda_2 t}) - e^{-\lambda_2 t} \bar{F}'(t)}{\bar{F}^2(t)}$$

$$= \frac{e^{-\lambda_2 t}}{\bar{F}(t)} [\lambda_2 - r(t)] .$$

$$P_2'(t) = - \frac{\bar{F}(t)(-\lambda_1 e^{-\lambda_1 t}) - e^{-\lambda_1 t} \bar{F}'(t)}{\bar{F}^2(t)}$$

$$= \frac{e^{-\lambda_1 t}}{\bar{F}(t)} [\lambda_1 - r(t)] .$$

$P_0(t)$ decreases in time from $P_0(0) = 1$ to $\lim_{t \rightarrow \infty} P_0(t) = 0$, and $P_0'(t) < 0$ for all t . $P_1(t)$ is increasing in time from $P_1(0) = 0$ to $\lim_{t \rightarrow \infty} P_1(t) = 1$, and $P_1'(t) > 0$ for all t .

$P_2(t)$ has a maximum at a time τ which can be found by substituting for $r(t)$ in $P_2'(t)$ and setting this expression equal to zero, i.e. when

$$\frac{(\lambda_1 - \lambda_2)e^{-(\lambda_1 + \lambda_2)t} + \lambda_2 e^{-(2\lambda_1 + \lambda_2)t}}{\bar{F}^2(t)} = 0.$$

Multiplying both sides by $e^{(\lambda_1 + \lambda_2)t}$ and combining terms gives

$$e^{-\lambda_1 t} = \frac{\lambda_2 - \lambda_1}{\lambda_2}$$

and

$$\tau = \frac{1}{\lambda_1} \ln \left(\frac{2}{\lambda_2 - \lambda_1} \right).$$

The value of $P_2(t)$ at maximum is

$$\frac{\lambda_2^2 - \lambda_1 \lambda_2 - (\lambda_2 - \lambda_1) \left(1 + \frac{\lambda_2}{\lambda_1} \right)}{\lambda_2^2 - \lambda_1 \lambda_2 + \lambda_2^{\lambda_2/\lambda_1} \lambda_1 (\lambda_2 - \lambda_1) - (\lambda_2 - \lambda_1)^{1 + (\lambda_2/\lambda_1)}},$$

and this is less than one for all $\lambda_2 > \lambda_1$.

As a consequence of the definition of the state probabilities and the requirement that

$$P_0(t) + P_1(t) + P_2(t) = 1,$$

the derivatives are related and

$$P_0'(t) + P_1'(t) + P_2'(t) = 0 .$$

Therefore, $P_2'(t) = -[P_0'(t) + P_1'(t)]$ and $P_2(t)$ increases when $-P_0'(t) > P_1'(t)$ and it decreases when $-P_0'(t) < P_1'(t)$. $P_2(t)$ increases as long as $P_0(t)$ decreases in time faster than $P_1(t)$ increases, reaches its maximum value when $-P_0'(t) = P_1'(t)$, and then decreases because $P_1(t)$ is increasing in time faster than $P_0(t)$ is decreasing.

A hazard rate may be thought of as a conditional instantaneous probability of failure at time t , given survival to time t . The hazard rate for the system, given that it is in one of its three states, is 0 , λ_1 , and λ_2 respectively. The system hazard rate may be written in terms of these hazard rates and the state probabilities as

$$r(t) = 0 \cdot P_0(t) + \lambda_1 P_1(t) + \lambda_2 P_2(t)$$

$$= 0 \cdot \frac{e^{-(\lambda_1 + \lambda_2)t}}{\bar{F}(t)} + \lambda_1 \frac{e^{-\lambda_1 t} e^{-(\lambda_1 + \lambda_2)t}}{\bar{F}(t)} + \lambda_2 \frac{e^{-\lambda_2 t} e^{-(\lambda_1 + \lambda_2)t}}{\bar{F}(t)}$$

which reduces to the expression previously given for $r(t)$.

The derivative of the system hazard rate with respect to time may be written as

$$r'(t) = \lambda_1 P_1'(t) + \lambda_2 P_2'(t) .$$

Then the condition for the maximum point of the hazard rate, $r(t) = 0$, requires

$$\lambda_1 P_1'(t) + \lambda_2 P_2'(t) = 0 \quad .$$

Substituting for $P_1'(t)$ and $P_2'(t)$ yields

$$\lambda_1 \frac{e^{-\lambda_2 t}}{\bar{F}(t)} [\lambda_2 - r(t)] + \lambda_2 \frac{e^{-\lambda_1 t}}{\bar{F}(t)} [\lambda_1 - r(t)] = 0 \quad .$$

Solving this for $r(t)$ gives its maximum value

$$r(t) = \frac{\lambda_1 \lambda_2 [e^{-\lambda_1 t} + e^{-\lambda_2 t}]}{\lambda_1 e^{-\lambda_2 t} + \lambda_2 e^{-\lambda_1 t}} \quad .$$

From $P_1'(t) = \frac{e^{-\lambda_2 t}}{\bar{F}(t)} [\lambda_2 - r(t)] > 0$ for all t , it is clear

that $r(t) < \lambda_2$ for all t . At its maximum value $r(t) > \lambda_1$ if and only if

$$\frac{\lambda_1 \lambda_2 [e^{-\lambda_1 t} + e^{-\lambda_2 t}]}{\lambda_1 e^{-\lambda_2 t} + \lambda_2 e^{-\lambda_1 t}} > \lambda_1 \quad ,$$

or

$$\lambda_1 \lambda_2 e^{-\lambda_1 t} + \lambda_1 \lambda_2 e^{-\lambda_2 t} > \lambda_1^2 e^{-\lambda_2 t} + \lambda_1 \lambda_1 e^{-\lambda_1 t} \quad ,$$

or

$$\lambda_1 \lambda_2 > \lambda_1^2 ,$$

or

$$\lambda_2 > \lambda_1 .$$

Therefore, at its maximum, $\lambda_1 < r(t) < \lambda_2$.

From $P_2'(t) = \frac{e^{-\lambda_1 t}}{\bar{F}(t)} [\lambda_1 - r(t)]$, it is clear that

$r(t) = \lambda_1$ at the time $\tau = \frac{1}{\lambda_1} \ln \left(\frac{\lambda_2}{\lambda_2 - \lambda_1} \right)$ when $P_2(t)$ reaches its maximum value. If $t > \tau$, then $r(t) > \lambda_1$. $r(t) > \lambda_1$ if and only if .

$$\lambda_1 P_1(t) + \lambda_2 P_2(t) > \lambda_1 .$$

or

$$\lambda_2 P_2(t) > \lambda_1 (1 - P_1(t)) ,$$

or

$$(\lambda_2 - \lambda_1) P_2(t) > \lambda_1 P_0(t) .$$

Since

$$P_2(t) = (e^{\lambda_1 t} - 1) P_0(t) ,$$

then $r(t) > \lambda_1$ if and only if

$$(\lambda_2 - \lambda_1)(e^{\lambda_1 t} - 1) > \lambda_1 ,$$

and for $t > \tau = \frac{1}{\lambda_1} \ln \left(\frac{\lambda_2}{\lambda_2 - \lambda_1} \right)$,

it follows that

$$\begin{aligned}
 (\lambda_2 - \lambda_1)(e^{\lambda_1 t} - 1) &> (\lambda_2 - \lambda_1)(e^{1[\frac{1}{\lambda_1} \ln(\frac{\lambda_2}{\lambda_2 - \lambda_1})] - 1}) \\
 &= (\lambda_2 - \lambda_1)(\frac{\lambda_2}{\lambda_2 - \lambda_1} - 1) \\
 &= \lambda_1
 \end{aligned}$$

As λ_2 approaches λ_1 in value τ becomes very large, and if $\lambda_1 = \lambda_2 = \lambda$, the system hazard rate $r(t)$ approaches λ asymptotically from below. For $\lambda_2 \gg \lambda_1$, τ becomes very small and the system hazard rate is greater than λ_1 almost instantly.

From $r'(t) = \lambda_1 P_1'(t) + \lambda_2 P_2'(t)$, $r(t)$ is increasing if and only if $\lambda_1 P_1'(t) + \lambda_2 P_2'(t) > 0$, and decreasing if and only if $\lambda_1 P_1'(t) + \lambda_2 P_2'(t) < 0$. The behavior of the system hazard rate is directly dependent upon the rates of change of the state probabilities $P_1(t)$ and $P_2(t)$. $r(t)$ increases as long as $P_1(t)$ and $P_2(t)$ increase and it continues to increase as long as $\lambda_1 P_1'(t) > |\lambda_2 P_2'(t)|$. For $\lambda_1 P_1'(t) < |\lambda_2 P_2'(t)|$, $r(t)$ decreases.

Figure 1 shows the state probabilities and the system hazard rate plotted as a function of time. As the failure rate of component two increases in relation to the failure rate of component one, these curves are "pushed towards the left", the system hazard rate increases rapidly, and its

maximum value increases. $P_0(t)$ decreases to zero very rapidly while $P_1(t)$ increases to one almost instantly. $P_2(t)$ becomes more peaked and it reaches its maximum value sooner.

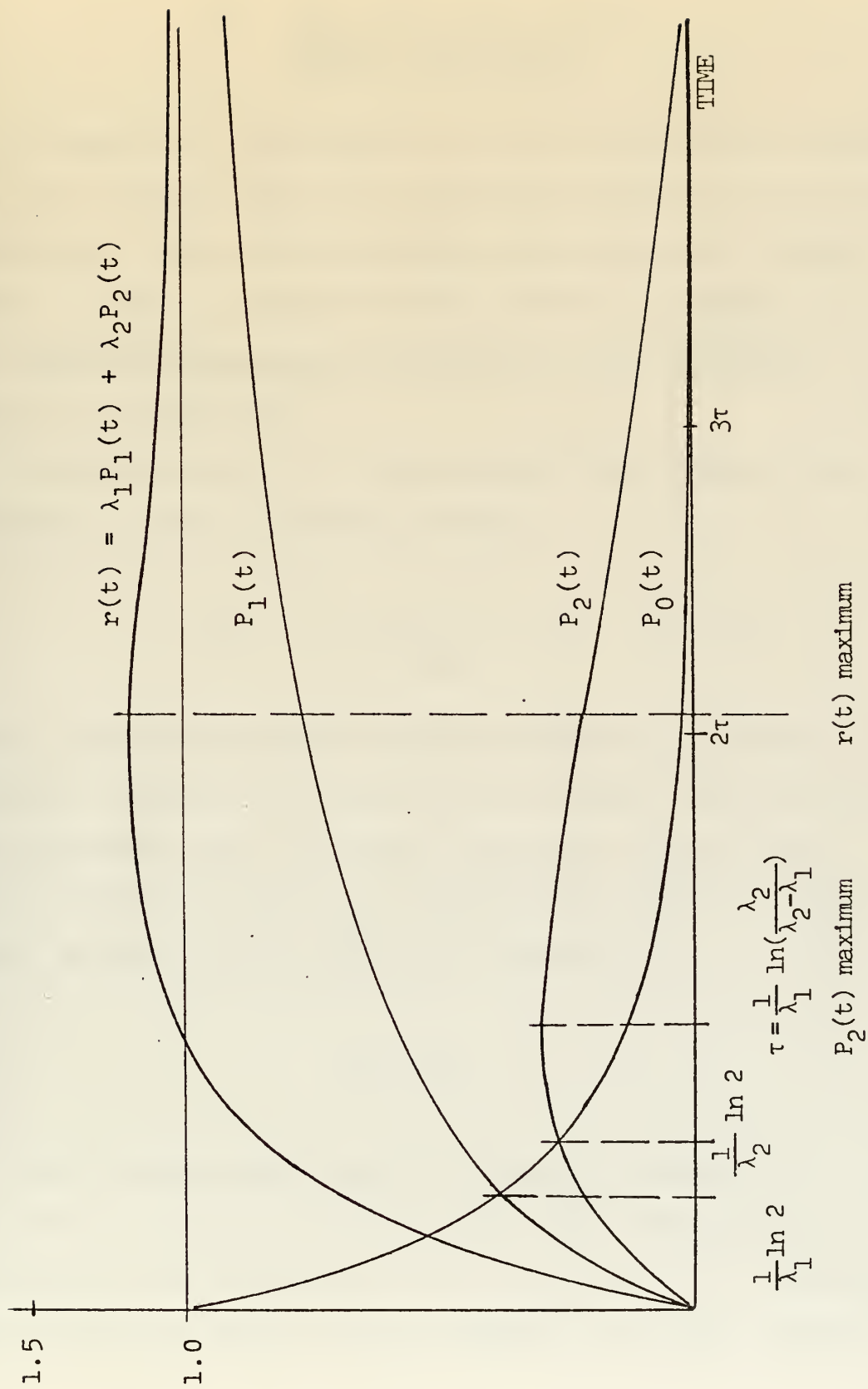


FIGURE 1. $r(t)$, $P_1(t)$, and $P_2(t)$ versus time. $\lambda_1=1$, $\lambda_2=1.5$.

IV. EMPIRICAL RELATIONSHIP BETWEEN TWO MAXIMA

An empirical investigation of the relationships between the state probabilities and the system hazard rate was also undertaken. The system equations were solved for various values of their parameters which permitted comparison of the resulting curves and verification of the analytic results of Section III.

The time at which the system hazard rate reaches its maximum, T_s , can be found by solving

$$\lambda_2^2 e^{-\lambda_1 t} + \lambda_1^2 e^{-\lambda_2 t} - (\lambda_1 - \lambda_2)^2 = 0$$

for t . While this can be solved numerically, a useful analytic solution for T_s has not been found. However, it was discovered that the time at which the system hazard rate reaches its maximum is approximately related to the time τ at which $P_2(t)$ reaches its maximum by

$$T_s \approx 2\tau.$$

From the numerical results, it appears that T_s is related to τ by some function of $\alpha = \frac{\lambda_2}{\lambda_1}$, the imbalance between the two components, i.e. $T_s = g(\alpha) \cdot \tau$. The exact form of $g(\alpha)$ is unknown but it appears to be some function

whose values are plotted in Figures 2, 3, and 4. Note that $g(\alpha)$ is never greater than 2.17, nor less than 2.0 for all $1.001 \leq \alpha \leq 100$, and for values of $\alpha > 15$, $g(\alpha) < 2.01$.

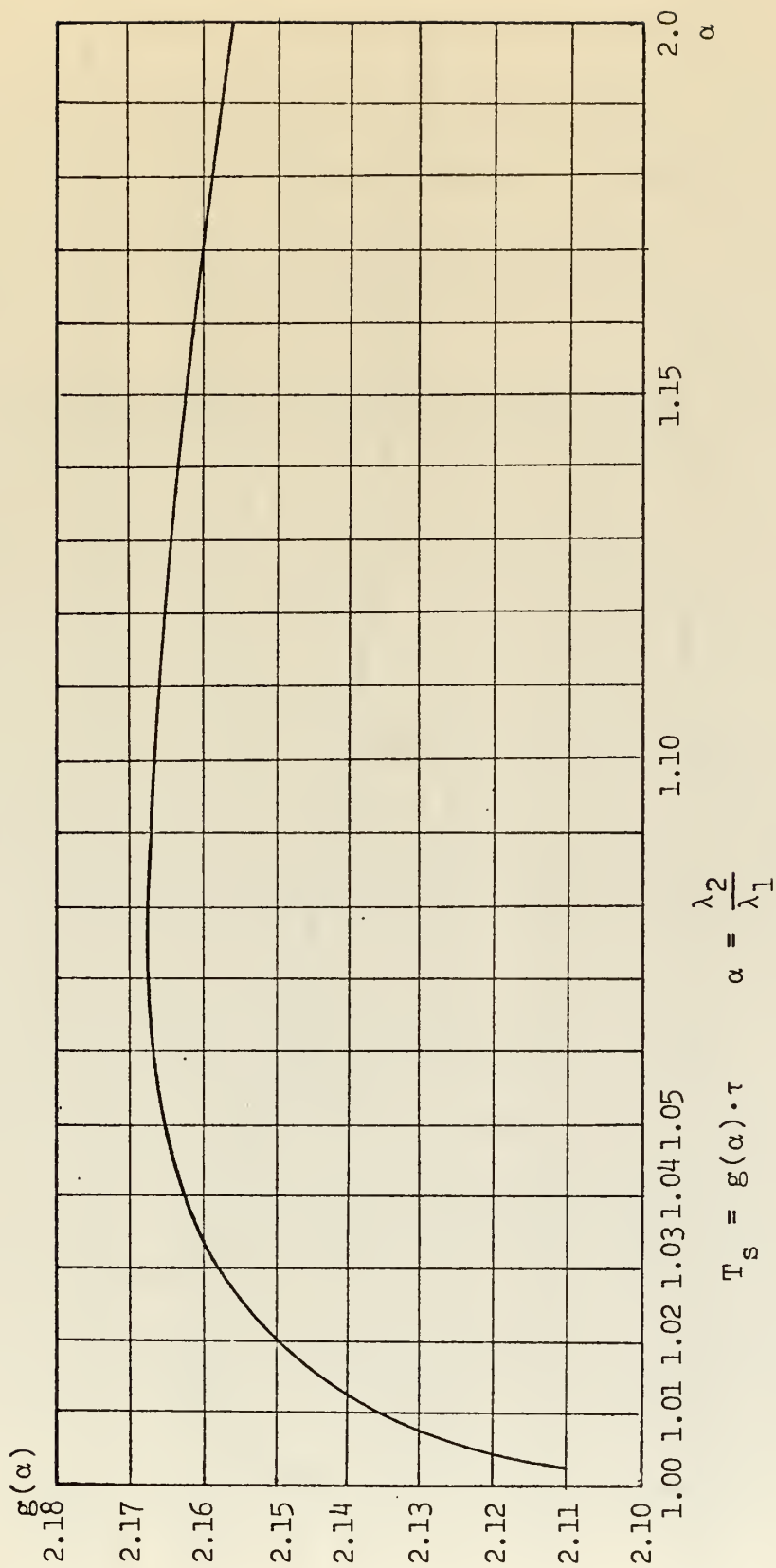


FIGURE 2.

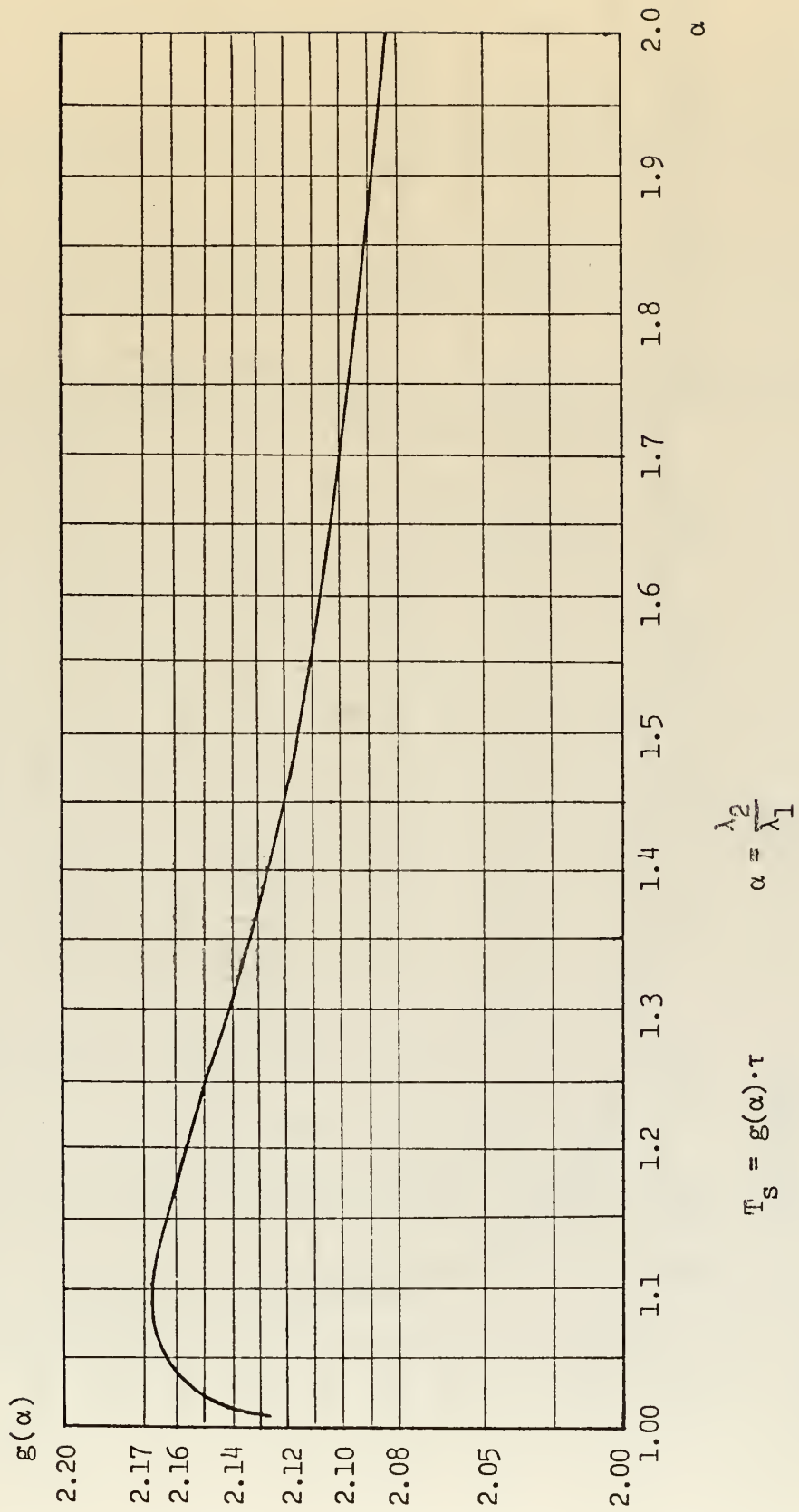


FIGURE 3

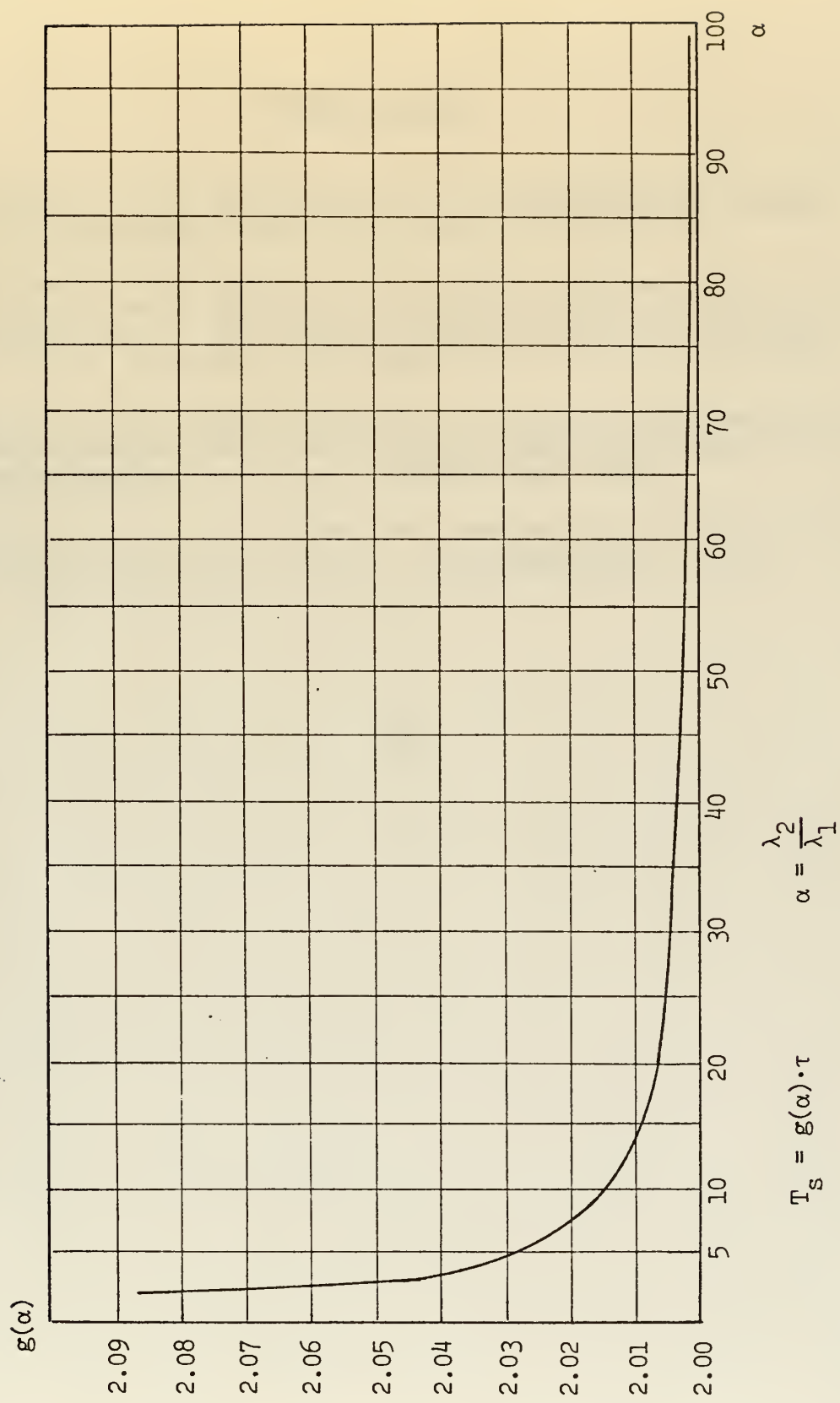


FIGURE 4

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